

## MECHANICS (C) UNIT 1

## TEST PAPER 9

Take  $g = 9.8 \text{ ms}^{-2}$  and give all answers correct to 3 significant figures where necessary.

- A particle  $P$  moves in a straight line with acceleration  $t - \frac{4}{t^2} \text{ ms}^{-2}$  at time  $t$  seconds, for  $t \geq 1$ .  
When  $t = 2$ ,  $P$  is instantaneously at rest.  
Calculate the magnitude of the velocity of  $P$  when  $t = 4$ . [5]
- A book rests on a rough desk-lid which is hinged at one end. When the lid is raised so that it makes an angle of  $15^\circ$  with the horizontal, the book is just on the point of sliding down a line of greatest slope. Modelling the book as a particle, find

  - the coefficient of friction between the book and the desk-lid, [2]
  - the acceleration with which the book starts to move if it is released from rest when the lid is inclined at  $20^\circ$  to the horizontal. [4]
- A jet of water issues from a cylindrical pipe with a circular cross-section of radius  $2.75 \text{ cm}$ . The water strikes a vertical wall at a speed of  $9 \text{ ms}^{-1}$ .  
Taking the density of water to be  $1000 \text{ kg m}^{-3}$ , calculate the momentum destroyed each second by the impact with the wall. State one modelling assumption that you have made. [6]
- A particle  $P$  is projected vertically upwards from ground level at time  $t = 0$  with speed  $20 \text{ ms}^{-1}$ . Two seconds later another particle  $Q$  is projected vertically upwards with speed  $30 \text{ ms}^{-1}$  from a point on the same horizontal ground.

  - Taking the upward direction as positive, write down expressions in terms of  $g$  and  $t$  for the velocities of  $P$  and of  $Q$  at time  $t$  seconds after  $P$  is projected. [3]
  - Find the value of  $t$  when both particles are moving with the same speed. [4]
- Two particles  $A$  and  $B$ , of mass  $1 \text{ kg}$  and  $m \text{ kg}$  respectively, where  $m > 1$ , are attached to the ends of a light inextensible string which passes over a small fixed smooth pulley. The particles are released from rest and move with the string taut and vertical.

  - Show that the acceleration of the system is equal to  $\frac{(m-1)g}{m+1}$ . [6]
  - Find the tension in the string, in terms of  $m$  and  $g$ , expressing your answer as a single algebraic fraction in its simplest form. [2]

When the system is released from rest, both particles are  $52.5 \text{ cm}$  above ground level and  $60 \text{ cm}$  below the level of the pulley.  $B$  hits the ground after half a second.

  - Find the value of  $m$ . [5]
  - Find the speed with which  $B$  hits the ground. [3]

**MECHANICS 1 (C) TEST PAPER 9 Page 2**

6. A particle starts from rest and accelerates at a uniform rate over a distance of 12 m. It then travels at a constant speed of  $u \text{ ms}^{-1}$  for a further 30 seconds. Finally it decelerates uniformly to rest at  $1.6 \text{ ms}^{-2}$ .

(i) Sketch the velocity-time graph for this motion. [3]

(ii) Show that the total time for which the particle is in motion is

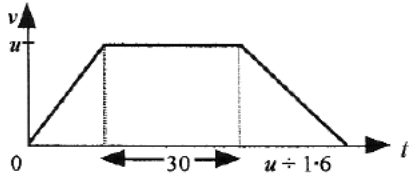
$$\left( \frac{5u}{8} + 30 + \frac{24}{u} \right) \text{ seconds.} \quad [5]$$

(iii) Find, in terms of  $u$ , the total distance travelled by the particle during the motion. [3]

(iv) Given that the total time for the motion is 39.5 seconds, show that  $5u^2 - 76u + 192 = 0$ . [3]

(v) Find the two possible values of  $u$  and the total distance travelled in each case. [6]

## MECHANICS 1 (C) TEST PAPER 9 : ANSWERS AND MARK SCHEME

1.  $v = \int a \, dt = \frac{1}{2}t^2 + \frac{4}{t} + c$   $v(2) = 0 : c = -4$  M1 A1 A1  
 $t = 4 : v = 8 + 1 - 4 = 5 \, \text{ms}^{-1}$  M1 A1 5
2. (i)  $R = W \cos 15^\circ$ ,  $\mu R = W \sin 15^\circ$   $\mu = \tan 15^\circ = 0.268$  M1 A1  
(ii) Acc down plane  $= g \sin 20^\circ - \mu g \cos 20^\circ = 0.0902g = 0.884 \, \text{ms}^{-2}$  M1 A1 M1 A1 6
3. Volume per second  $= \pi(0.0275)^2 \times 9 = 0.02138 \, \text{m}^3$  M1 A1  
having mass  $21.38 \, \text{kg}$  Momentum  $= mv = 21.38 \times 9 = 192 \, \text{Ns}$  A1 M1 A1  
Assumed water moves horizontally, does not rebound, etc. B1 6
4. (i)  $v_P = 20 - 9.8t$   $v_Q = 30 - 9.8(t - 2)$  B1 M1 A1  
(ii) Equal speeds when  $v_P = v_Q$  (never) or  $v_P = -v_Q$  : M1  
 $20 - 9.8t = 9.8t - 19.6 - 30$   $19.6t = 69.6$   $t = 3.55$  M1 A1 A1 7
5. (i)  $mg - T = ma$ ,  $T - g = a$  Add :  $mg - g = ma + a$  M1 A1 A1 M1  
 $g(m - 1) = a(m + 1)$   $a = \frac{(m-1)g}{m+1}$  A1 A1  
(ii)  $T = a + g = \frac{2mg}{m+1}$  M1 A1  
(iii)  $0.525 = \frac{1}{2} \frac{(m-1)g}{m+1} \left(\frac{1}{2}\right)^2$   $\frac{m-1}{m+1} = \frac{3}{7}$   $7m - 7 = 3m + 3$  M1 A1 A1  
 $4m = 10$   $m = 2.5$  M1 A1  
(iv) Then  $a = \frac{3g}{7} = 4.2$   $v = at = 0.5a = 2.1 \, \text{ms}^{-1}$  B1 M1 A1 16
6. (i)  B3  
(ii) Let three times be  $t_1, t_2, t_3$   $\frac{1}{2}ut_1 = 12$   $t_1 = \frac{24}{u}$  M1 A1  
 $t_2 = 30$   $u + t_3 = 1.6$   $t_3 = u + 1.6 = \frac{5u}{8}$  Hence result M1 A1 A1  
(iii) Distance = sum of areas  $= 12 + 30u + \frac{1}{2}u \frac{5u}{8} = \frac{5u^2}{16} + 30u + 12$  M1 A1 A1  
(iv)  $\frac{5u}{8} + 30 + \frac{24}{u} = 39.5$   $\times 8u : 5u^2 + 240u + 192 = 316u$  M1 A1  
 $5u^2 - 76u + 192 = 0$  A1  
(v)  $(5u - 16)(u - 12) = 0$   $u = 3.2$  or  $u = 12$  M1 A1 A1  
When  $u = 3.2$ , dist.  $= 111 \, \text{m}$  When  $u = 12$ , dist.  $= 417 \, \text{m}$  M1 A1 A1 20